

MAS212 Linear Algebra I 2007-2008

Coursework 7

Please hand in your solutions of the **starred** exercises by **2pm on Friday 23 November** using the [blue Linear Algebra I Collection Box](#) on the Ground Floor.

Don't forget to put your **name** and **student number** on your solutions, and to **staple** them.

Exercise* 1. (2 marks) Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{b}_1 = \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}, \mathbf{b}_2 = \begin{pmatrix} -3 \\ 1 \\ 12 \end{pmatrix}, \mathbf{b}_3 = \begin{pmatrix} 2 \\ 0 \\ -7 \end{pmatrix} \quad \text{and} \quad \mathbf{d}_1 = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}, \mathbf{d}_2 = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}, \mathbf{d}_3 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}.$$

Show that $[\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3]$ and $[\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3]$ are ordered bases for $\mathbb{R}^3$.

Exercise* 2. (2 marks) Let $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3 \in P_2$ be given by

$$\mathbf{p}_1(t) = 1 - t^2, \quad \mathbf{p}_2(t) = t, \quad \mathbf{p}_3(t) = t + 2t^2.$$

- (a) Use the answer to Exercise 2 (b) of Coursework 6 to deduce that $\mathcal{B} = [\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3]$ is an ordered basis for P_2 .
- (b) Let $\mathbf{p} \in P_2$ be given by $\mathbf{p}(t) = 1 - t + 3t^2$. Find the coordinates of $\mathbf{p}$ relative to $\mathcal{B}$.

Exercise* 3. (4 marks) Let $\mathcal{B} = [\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3]$ and $\mathcal{D} = [\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3]$ be the ordered bases given in Exercise 1.

- (a) What is the transition matrix S_1 from $\mathcal{D}$ to the standard basis?
- (b) Determine the transition matrix S_2 from the standard basis to $\mathcal{B}$, and, hence, determine the transition matrix from $\mathcal{D}$ to $\mathcal{B}$.
- (c) If the $\mathcal{D}$ -coordinate vector of $\mathbf{x} \in \mathbb{R}^3$ is $(3, 1, -2)^T$, what is the $\mathcal{B}$ -coordinate vector of $\mathbf{x}$?

Exercise 4. Let $A \in \mathbb{R}^{m \times n}$. Show the following: the system $A\mathbf{x} = \mathbf{b}$ has at most one solution for every $\mathbf{b} \in \mathbb{R}^m$ if and only if the column vectors of A are linearly independent.

Exercise* 5. (6 marks) Let

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 & 3 \\ 1 & 2 & -1 & 0 & 4 \\ 2 & 4 & -3 & 1 & 7 \end{pmatrix}.$$

Find bases for $\text{row}(A)$, $\text{col}(A)$ and $N(A)$. Determine the rank and nullity of A , and verify that the Rank-Nullity Theorem holds for the matrix A .

Exercise* 6. (2 marks) Let $A \in \mathbb{R}^{n \times n}$. Use the Rank-Nullity Theorem to show the following: A is invertible if and only $\text{rank } A = n$.

Exercise* 7. (4 marks) Determine which of the following are linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^3$:

- (a) $L(\mathbf{x}) = (1, x_2, x_1)^T$,
- (b) $L(\mathbf{x}) = (x_1, 0, x_2)^T$,
- (c) $L(\mathbf{x}) = (x_1, x_2, x_1 - x_2)^T$,
- (d) $L(\mathbf{x}) = (x_1, x_2^2, 0)^T$.